

# Improvement of the Aerodynamic Tunnel used by João de Deus & Filhos S.A.

Eduardo Queiroga Dunões

eduardodunoes@tecnico.ulisboa.pt

Instituto Superior Técnico, Universidade de Lisboa, Portugal

December 2018

## Abstract

The objective of this collaboration with JDEUS was the improvement of the test bench which the company uses to assess the performance of an intercooler, with focus on finding a way to measure with accuracy the air flow velocity passing externally across the tested intercooler.

Trough the analysis of the installation and the type of tests there performed, it was concluded that the best place to measure the air flow velocity should be before the flow reaches the intercooler. Two measurement methods were evaluated: computation through dynamic pressure and resorting to hot-element anemometers. The application of low-cost hot-element anemometers was evaluated as a possible solution. In order to be used with confidence, the anemometers were experimented and calibrated in a wind tunnel available at Instituto Superior Técnico.

It has been verified that the application of a well design contraction, which directs the flow at the entrance of the test section of the intercooler, is essential, allowing the flow velocity to be measure upstream the intercooler.

The contraction was designed and built, and the two measurement methods were validated experimentally. It was concluded that the best solution to measure the air flow velocity accurately will be a combination of the two methods: hot-element anemometer for low velocities and dynamic pressure for high velocities.

**Keywords:** Wind tunnel, Air flow velocity, Intercooler, Dynamic pressure, Hot-wire anemometer

---

## 1. Introduction

Nowadays, the search for a continuous improvement in the automotive industry, together with the high competitiveness between manufacturers, requires a rigorous treatment of the various components of a car.

João de Deus & Filhos, S.A., a company that develops and produces intercoolers, has an aerodynamic tunnel to test them, with the purpose of rigorously understand their behavior, and guarantee the specification imposed by the company's customers. Over the years that installation undergone some changes which, by favoring certain test parameters, have made other unfavorable. The improvement of this test bench is of great interest to the company because it allows to guarantee with rigor the conditions of operation of the products that they sell.

In this work a study of the installation is made, focusing on the main problem that JDEUS exposes, which is the rigorous measurement of the air flow that

crosses the intercooler externally. The importance of a correct flow measurement is due to the fact that JDEUS' customers require that the intercooler's efficiency, power and external pressure drop must be presented as a function of the flow or speed of the air.

After analyzing a possible solution to the problem, it is validates experimentally.

## 2. Intercooler's test benches

Intercooler's test benches are wind tunnels that allows the evaluation of their performance.

The tunnels developed to study intercoolers and radiators have some variations between their configuration, however, in addition to certifying the control and monitoring of all variables involved, they must all ensure two particularities: first, they must ensure that the external air flow incident to the intercooler is normal to the front face of the intercooler with an approximately uniform velocity profile, and secondly it should allow the testing of intercoolers and radiators with

various dimensions. No matter the type of tunnel the working principle is, frequently, similar between them (Figure 1.1).

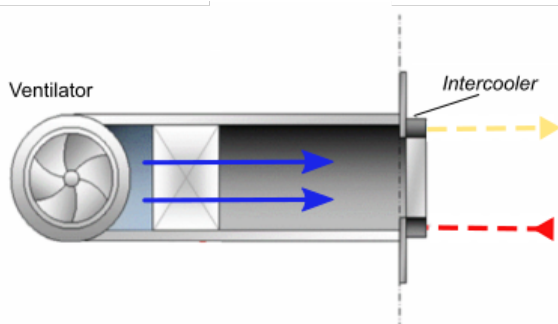


Figure 1 - Test bench working principle.

In all the configuration of tunnels investigated, the way of coupling intercoolers of different sizes is done by adjusting only the test aperture according to the size of the piece to be tested. However, this solution is not ideal because, since the channel does not have the same dimensions of the piece, it does not guarantee a completely frontal flow in the face of the intercooler, mainly in its periphery. This phenomenon is undesirable as it causes an additional pressure loss in the intercooler.

The JDEUS' test bench was developed with a mechanism that guarantees a fully frontal external air flow with an approximately uniform velocity profile. The characteristics of this mechanism will be explained latter.

## 2.1. Flow Measurement

In the case of installations as those described above, it is common to use flowmeters in the ducts of the tunnel. The use of this kind of flowmeters requires a fully developed flow, which requires a large rectilinear duct, that is not always possible. Another disadvantage in the use of flowmeters is that they do not guarantee good accuracy at all flow rates. The solution is the installation of conduits in parallel, each one destined to measure a range of flows.

The flow measure through dynamic pressure or anemometers, fan or hot-wire, can also be good alternatives, although, to achieve good measures it is necessary to ensure that the measurement is made in a section where the flow has an approximately uniform velocity profile and a reduced level of turbulence. In order to ensure a uniform and low turbulence velocity profile it is necessary to apply flow conditioners. These may be honeycombs structures, nets or contractions. Due to the configuration of the JDEUS' test bench, a contraction as a flow conditioner is the most viable alternative if we want to measure the flow velocity with one of the last mentioned methods.

## 3. Test Bench Description

The JDEUS' test bench is divided into two separate divisions, the test room and the machine room. The test room contains the stabilization chamber and the test section, and the machine room is where the ventilator, responsible for the air flow, the ducts connecting the various components, and the whole system responsible for promoting the internal flow that passes through the intercooler. In a third division, called the control room, is located the whole system of control and acquisition of the test variables (Figure 2).

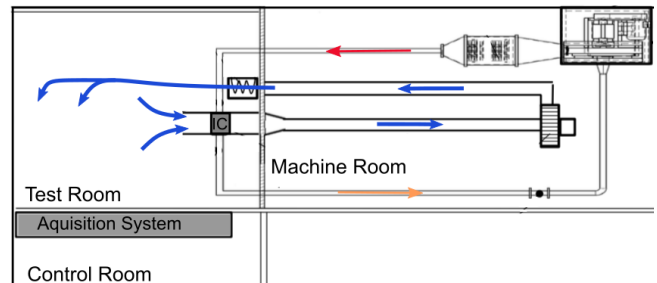


Figure 2 - Test bench Scheme.

Since the company develops and manufactures intercoolers and radiators for various brands and various car models, the geometries of the parts produced by them vary widely. In order to enable the tests of geometries with the most varied sizes, JDEUS developed the test section of the tunnel with a versatile mechanism capable of easily adjust to the size of any intercooler with a rectangular section (Figure 3).

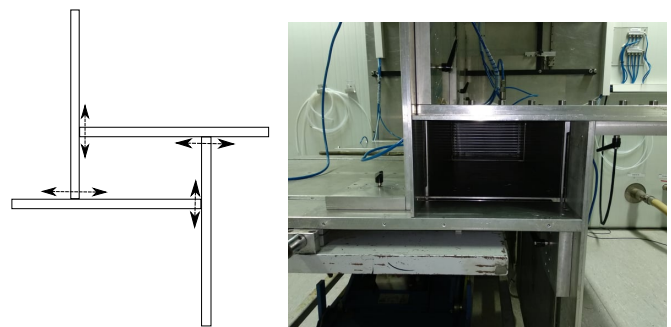


Figure 3 - Test section mechanism.

The configuration of the test section was design so that the intercooler to be test is subjected to a frontal, parallel and approximately uniform flow.

The test section mechanism is composed by two segments, each consisting of four plates. These plates slide between them through rails to allow the test section adjustment, depending on the intercooler's dimensions. One of the sets of plates is downstream and the other is upstream the intercooler, allowing the adjustment according to the its thickness. This set allows to create a

constant section duct, equal to the frontal area of the intercooler.

### 3.2. Problem Characterization

The tests performed in this tunnel allow to make thermodynamic balances and calculate the efficiency, power and external loss of the intercoolers as a function of the external air mass flow rate.

To determine the mass flow rate, one can measure the velocity and density in a section of known area. When there is heat exchange, the density is only well known at the entrance. Therefore, the mass flow rate can be controlled by the velocity at the inlet.

Currently the company has difficulty in measuring with the desired precision the flow of air crossing the testes intercoolers. Some studies of the instalation suggest that the average air flow velcity, that reach the frontal section of the intercooler, is approximately  $\frac{3}{4}$  of the velocity measured with a fan anemometer at the conduit inlet. This difference is explained by the inlet duct being plates with sharp edges. The flow separates near the inlet, creating a narrowing of the main flow section and, consequently an accelaration [1] (Figure 4). The  $\frac{3}{4}$  corrective factor currenty applied has some disadvantages because its not absolute, depending on the cross-sectional geometry of the intercooler. The entrance of the duct is also not the best place to measure flow speed, since it is not uniform transversely and oscilates significantly.

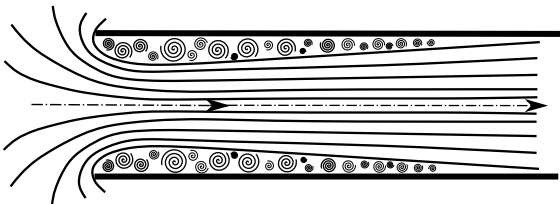


Figure 4 - Flow phenomena entering a sharp edge duct.

Another relevant deficiency is that the fan anemometer is not fixed. The air flow readings are only made before the test, without hot internal airflow in the intercooler, and subsequently relates to the intercooler's pressure drop.

The total pressure loss in the external air circuit of an intercooler,  $\Delta P_{intercooler}$ , is given by:

$$\Delta P_{intercooler} = K \frac{\rho \cdot V^2}{2} = K \frac{G^2}{2 \cdot \rho} \quad (1)$$

The coefficient  $K$  of total pressure loss of the external flow is approximately constant, independent of the Reynolds number.

For the case where the external air density is approximately constant it is possible to assume that  $\Delta P \propto V_m^2 \approx V^2$ , where  $V$  is the air flow velocity that impinges on the exchanger. Thus, the air flow velocity is directly related to the external in the part, as assumed in the current tests made:  $V = f(\Delta P_{intercooler})$ .

Currently, the mass flow of the external air is controlled through the total pressure loss along the intercooler but, when occurs heat exchanges the density varies and also the velocity. Thus, the same pressure loss corresponds to different velocities at the entrance and, therefore, to different mass flow rates.

Therefore, finding an alternative that guarantees a correct air flow measurement becomes a priority.

### 4. Solution Strategy

The solution of the problem described above has an important requirement: not to make a significant change in the structure of the test bench that imposes the paralysis of the test bench over a long period.

A solution to be considered would be to install a flowmeter in the conduit connecting the test section to the fan, which has in fact already been an alternative used. However, this solution presents problems. The connection of the test zone to the conduit does not guarantee a perfect tightness and, therefore, air infiltrations in the circuit resulting in excess measures. Even in cases where infiltration could be avoided, a flowmeter in the pipeline of the current facility is unable to measure low speeds accurately. Reducing the duct section, suitable at low speeds, is out of the question, as it would make it impossible to perform tests at higher speeds due to throat loss. The solution would be the implementation of a second conduit in parallel, which makes it possible to read the lower speeds. This solution would require significant changes to the installation, including control systems.

The alternative proposed in this work is to measure the flow velocity before the intercooler. Two methods were analyzed: speed measurement through dynamic pressure and through hot wire anemometer.

Due to the current configuration of the test section inlet, also these measurement methods need some attention, so that the results obtained are reliable. It has been concluded that for either method, it will be necessary to install a contraction at the inlet of the conduit to ensure alignment of the flow and reduction of disturbances.

#### 4.1. Velocity through Dynamic Pressure

The bench is equipped with a pressure sensor system in the stabilization chamber and in a section of the conduit, which makes it possible to estimate the velocity

from the mechanical energy balance. Neglecting the heat exchanges, with a control volume delimited by an inlet section inside the stabilization chamber, and an outlet section inside the conduit, under stationary conditions it is possible to write:

$$V = \sqrt{\frac{2 \cdot \Delta P_{entrada}}{(1 + \xi) \cdot \rho_{ar}}} \quad (2)$$

The system pressure loss  $\Delta P_{entrada}$  is a local loss and is related to the abrupt reduction of the flow cross-section (Figure 4). This local loss is proportional to the dynamic pressure multiplying by a coefficient. The pressure loss coefficient  $\xi$  varies from situation to situation. According to Idel'chik [1], the loss coefficient associated with the entrance of a flow in a duct of constant rectangular section with sharp edges, with a ratio between the sides of  $0,6 \leq l/h \leq 1,7$ , depends on two characteristics: the relative thickness,  $\delta/D_h$ , of the conduit wall, and the relative distance,  $b/D_h$ , from the conduit entry section to the wall where it is mounted

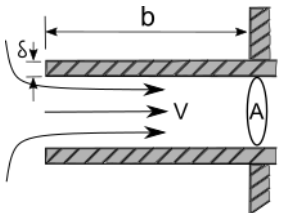


Figure 5 - Sharp edge inlet.

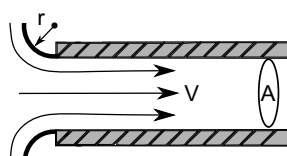


Figure 6 - Bell mouth inlet.

For the case of an inlet bell mouth, with a relative radius of curvature  $r/D_h = 0,2$ , the pressure loss coefficient can be reduced to about 0,03 [1]. Although, it is expected that a gradual entry will be even more favorable, with a pressure loss of almost zero at the inlet.

## 4.2. Velocity through Hot-Wire

The velocity measured through the hot wire anemometry is based on the heat transfer between the sensor and the surrounding fluid. The sensor consists of a wire/metal element heated by an electric current (Joule effect), which in turn is cooled by the incident flow, predominating forced convection [2]. In order to maintain the temperature of the element, through an electronic control circuit, an increase of current is supplied to the element. This current  $E$  is related to the speed  $U$  according to the law given by King [3]:

$$E^2 = A + BU^n \quad (3)$$

An advantage of this type of anemometer over others is that they can be quite small and do not disturb the flow in question. On the other hand, the hot-wire/element anemometer is a good option as it allows good accuracy at low speeds. This type of anemometer is very sensitive to variations in flow, so the need to insert it in a flow with low turbulence levels and a velocity profile as uniform as possible is important, which can be possible in the inlet of the installation as long as the contraction is well designed.

In a uniform flow it would be sufficient to measure the velocity at one point to calculate the average flow but a multi-sensor measurement introduces redundancy and increases accuracy.

Due to the high cost of hot-wire/element anemometers in the market, which guarantee a good precision of measurements, and to the fact that the installation of more than one apparatus is idealized, it was studied, as an alternative, the use of low cost anemometers, with a price ten times lower than a conventional anemometer.

The hot-element sensor used, named Wind Sensor Rev. P. (Figure 7), is a low cost anemometer developed for electronic projects.

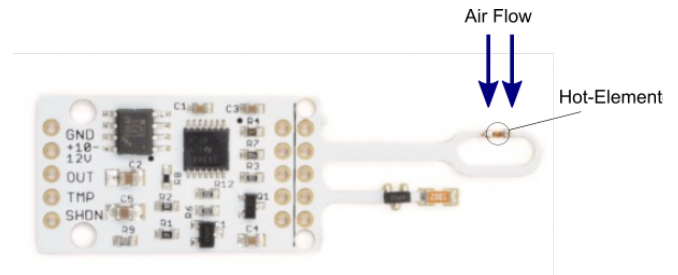


Figure 7 - Wind Sensor Rev. P.

As an acquisition system of the signal from the hot-element anemometer, an Arduino-UNO board, with 10 bits resolution was used.

A calibration of the anemometer is needed to establish a relationship between the anemometer output and the flow velocity. This relationship is established by exposing the sensor to a known set of velocities,  $U$ , and hence recording the respective voltage values,  $E$ . The curve of the points  $(E, U)$  represents the function to be used to convert the voltage signal into velocity. Although these anemometers potentially have low associated uncertainty, especially at low speeds, this is totally dependent on the uncertainty associated with the calibration process. Since a precise calibration tunnel was not available, a preliminary calibration was carried out in a tunnel of Instituto Superior Técnico.

### 4.3. Contraction Development

The design of a contraction that is applicable on the JDEUS' test bench has to meet certain requirements. First it must have a rectangular section, just like the conduit. The contraction has to adapt to the different sizes of the parts to be studied, as does the rest of the mechanism of the test section. Another point to consider is the contraction dimensions, which cannot be too large, so it won't difficult the handling and complicate the tests' preparation.

Due to the variable sided rectangular geometry it is not possible to use a common contraction, such as those referred in the literature [4 to 8]. Thus, the design of the contraction was based on a profile with a function studied by [9], which ensures that a practically uniform velocity profile is achieved in the narrow section with no curvature effects, resulting:

$$y = 0,6 \cdot x \cdot [\ln(0,6 \cdot x) + 1,12] \quad (4)$$

Where  $y$  is the height coordinate and  $x$  the longitudinal coordinate.

In order to avoid a possible separation of the boundary layer at the edge of the inlet, a curved flange was added.

All the manufacture, except for some finishing touches of the corners, was made by us in the Mechanical Engineering workshops of Instituto Superior Técnico.

The contraction was manufactured in four aluminum plates with a thickness of 2.5 mm. The plates went through several phases until reaching the final product: calendaring, bending and cutting.

The fitting of the plates in the installation was made with screws. After fitting it in the JDEUS' tunnel some adjustments were made so that the four plates constituting the contraction were all in harmony with each other, facilitating the adaptation to the desired section (Figure 8).

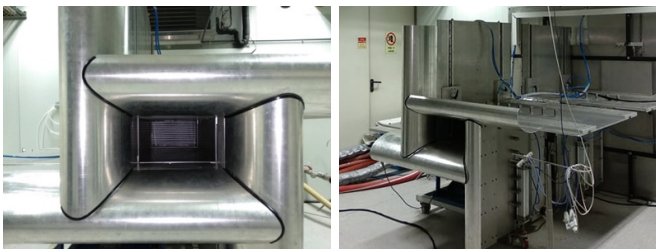


Figure 8 - Final configuration of the contraction.

### 5. Experimental Procedure

In this section is discussed the hot-element anemometer calibration process, however its main focus is the experimental analysis of the different inlet

configurations and the different flow measurement methods mentioned above.

As a first approach the tests were performed without the presence of internal air flow inside the intercooler. Subsequently complete tests were performed, with external and internal flow, in order to consolidate the results.

Only one intercooler was studied in depth.

#### 5.1. Hot-Element Calibration

It was intended to calibrate the anemometers in a wind tunnel projected specifically to calibrate this type of sensors. Although, as the projected tunnel was not available, a preliminary calibration of the anemometers was carried out in a wind tunnel of Instituto Superior Técnico. This calibration was carried out in the wind tunnel installed in the Fluid Mechanics laboratory, in the Mechanics IV building of Instituto Superior Técnico, in order to test the data acquisition.

The tunnel is equipped with a total Pitot tube within the flow, and static pressure holes at the exit of the contraction, both connected to an inclined tube manometer, allowing the calculus of the flow velocity at the exit.

For each velocity point calculated by the difference between the static pressure and the total pressure, measured with an alcohol column manometer, the recorded voltage values were averaged. The 80 measured points were imported into the Matlab software in order to plot the calibration curve. Through the toolbox "Curve Fitting Tool", available in Matlab, it was possible to draw a function that related the three variables involved, voltage, speed and temperature. Thus, the calibration curve shown in Figure 9 was obtained:

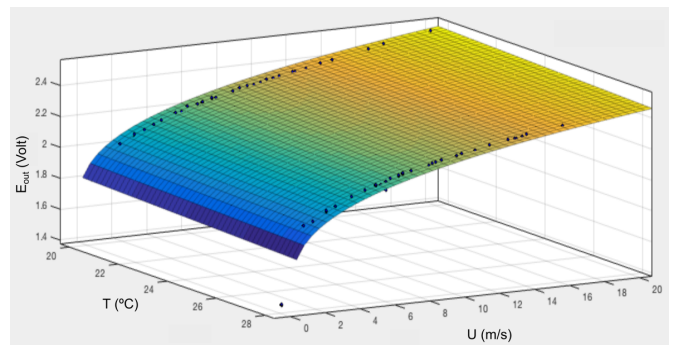


Figure 9 - Calibration function obtain in Matlab.

The calibration curve relates the anemometer voltage to the temperature, through a linear parameter, and to the flow velocity through a power parameter, which resembles the function (3) of King's law:

$$E_{out} = 1,577 - 0,005043 \cdot T + 0,430 \cdot U^{0,2872} \quad (5)$$

Rearranging equation (5) in order to obtain the velocity as a function of the voltage and temperature, the following transfer function was obtained:

$$U = \left[ \frac{(E_{out} - 1,577 + 0,005043 \cdot T)}{0,4305} \right]^{1/0,2872} \quad (6)$$

Although the resolution of the voltage signal ( $\cong 0.005$  Volt) is low, the fact that the transfer function is not linear causes the resolution to decrease as the speed increases. This resolution of the anemometer can be improved with an acquisition system with a higher resolution. A 12 bits acquisition system would be sufficient for the resolution of the anemometer in terms of velocity to always be less than 0.1 m/s for the range of values to be measured (2 - 14 m/s).

## 5.2. "Cold" Tests

The "cold" tests are performed in the absence of internal flow in the intercooler. These tests not only facilitate the initial approach to the problem, but are also done in this way for safety reasons. To do the speed acquisition with the hot-element anemometer it is necessary to be inside the room, which would not be safe in the event of a hose, with internal air flow at high temperature and pressure, bursting.

In this chapter is studied the initial situation with only the flat plates, with no contraction at the entrance of the conduit (Figure 10-a), the situation with a cylindrical profile at the entrance of 25 mm radius (Figure 10-b), and the situation with the logarithmic profile contraction, projected (Figure 10-c). The entrance with cylindrical profile is an alternative that had already been suggested and constructed, but that never had practical application, and, for the sake of validation and comparison, was also studied.

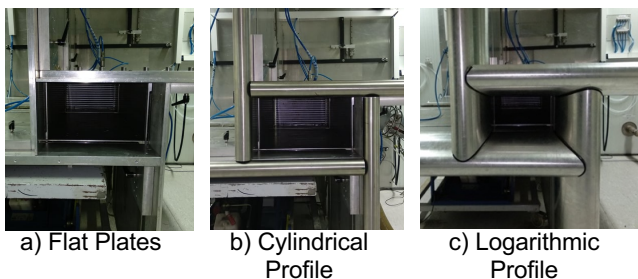


Figure 10 – Inlet configurations.

At first, the velocity was measured with the fan anemometer at the entrance of the tunnel only with the flat plates, which is then related to the pressure drop in

the intercooler, allowing the velocity to be controlled from the pressure loss (discussed in section 3.2).

During the tests, where the speed is controlled by the pressure loss, the values obtained by a pressure transducer and by the temperature and pressure sensors are acquired. For each speed five points were recorded when the speed was stabilized. The velocity was then calculated through equation (2) and averaged over the five points.

The coefficient of pressure loss  $\xi$ , required in equation (2), depends on each type of input configuration in the duct and can be obtain in [1]. For the flat plates  $\xi = 0,0535$ , and for the cylindrical profile  $\xi = 0,131$ . The situation with the logarithmic profile, the pressure drop coefficient is considered zero and the velocity was also recorded through the hot element anemometer located is the center of the de entrance section.

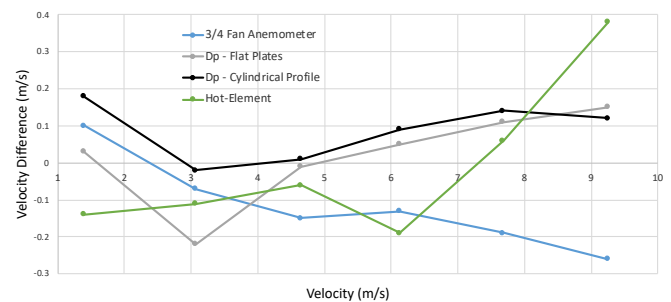


Figure 11 – Difference of the various velocity measurements comparing with the velocity measured by the dynamic pressure in logarithmic profile test.

The similarity of the values suggests that the logarithmic contraction provides an approximately uniform velocity profile (Figure 11). This was confirmed by crossing the section with the hot-element anemometer, measuring various points (Figure 12).

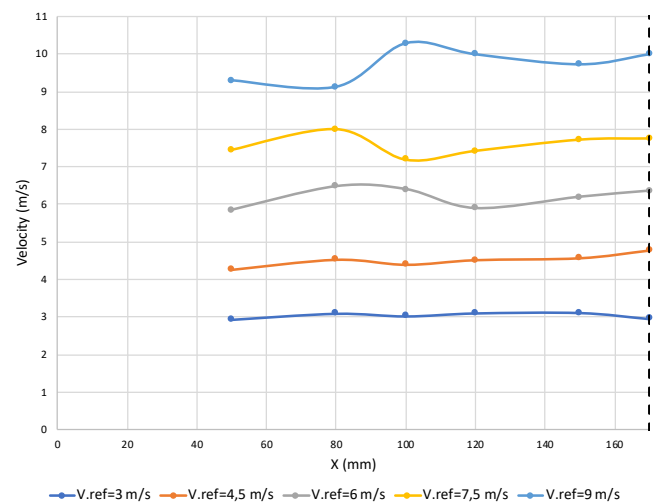


Figure 12 – Approximated velocity profile obtained.

### 5.3 Complete Test

A complete test in this test bench is one that has internal flow through the intercooler and, therefore, heat exchanges and temperature variation.

In this chapter we compare a test made in the primitive installation, with only the flat plates (Figure 10-a), with an assay in which the input has a logarithmic profile (Figure 10-c). In both cases, for safety reasons, it was only possible to measure velocity through the difference between the static pressure in the duct walls and the total pressure, which is the atmospheric pressure of the test room. The density was calculated from the absolute temperature and pressure.

The test in the primitive installation consists of:

- Measurement of the speed at the entrance with the fan anemometer and the external pressure loss with the intercooler at room temperature (without internal flow);
- Application of a correction factor of 3/4 speed and obtaining the relation between velocity and pressure loss.
- Intercooler test with internal flow rate, adjusting the external flow in order to restore the external pressure loss, according to the relation obtained previously.
- After the test, the inlet velocity is calculated by the difference between the total and static pressure according to equation (2) with  $\xi = 0,0535$ .

The test with the logarithmic profile mouted consists of:

- Intercooler test with internal flow rate, adjusting the external flow rate in order to obtain the average external flow rate required at the inlet, measured from the difference between the stagnation pressure in the room and the static pressure in the walls of the inlet channel. This corresponds to using equation (2) with  $\xi = 0$ .

The tests were performed with an internal flow rate of 200 and 600 kg/h, at a temperature of 180°C.

In the case of the common test in the primitive installation, where the average velocity of the external flow is controlled by the intercooler head loss, what is maintained is not the velocity at entry but a kind of average velocity along the part, defined as that which produces equal loss of load. In this way, the variation of the internal flow rate and the heat exchanges influence the measurements, since, in order to maintain the same pressure drop, it is necessary to change the speed at the entrance.

It is possible to verify that both the velocity results at the entrance of the duct, with the primitive installation and with the contraction mounted, are in conformity

when calculated through the dynamic pressure (Figure 13). The average velocity measured at the inlet corresponding to an equal pressure drop in an isothermal test shows higher values, which deviate from the input velocity values measured by dynamic pressure.

In tests with internal flow the density varies and also the velocity. Thus, the same head loss corresponds to a different inlet velocity than in a test where there is no internal flow.

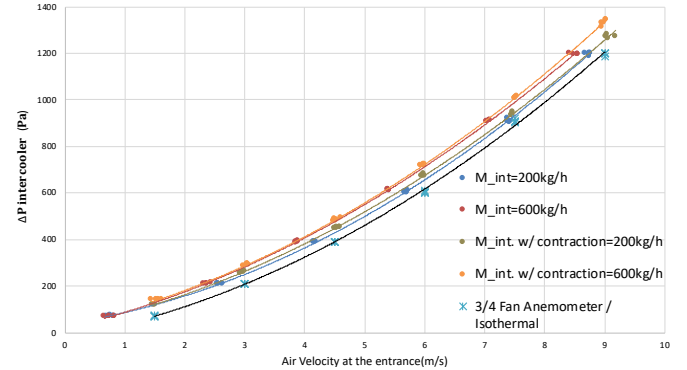


Figure 13 - Pressure loss as a function of the input velocity calculated by the dynamic pressure and the pressure drop of the primitive configuration ( $\xi = 0,0535$ ); input velocity calculated through dynamic pressure without pressure loss ( $\xi = 0$ ) in the plant equipped with the new logarithmic contraction and velocity corresponding to an equal loss of charge in an isothermal test.

This error of measurement of the average velocity at the entrance of the conduit will influence when evaluating the efficiency of the intercooler (Figure 14). The efficiency error is higher for low air velocities outside the inlet and for higher internal air flow rates, reaching a 13.3% error in the most extreme case. For the higher speeds, from 4.5 m/s, the speed error does not have as much impact on the efficiency of the intercooler, presenting at most one error of 2.6%, which corresponds to the external air velocity at the entrance of 4.5 m/s and an internal air flow of 600 kg/h.

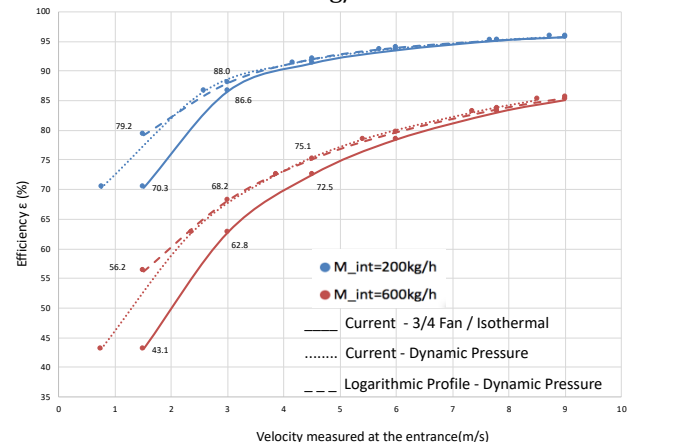


Figure 14 - Efficiency of the intercooler as a function of the velocity corresponding to an equal pressure drop in an isothermal test, air velocity external to the inlet of the duct

calculated by the dynamic pressure and pressure drop of the primitive installation ( $\xi = 0.535$ ) and velocity at input calculated through the dynamic pressure and without loss of load ( $\xi = 0$ ) in the installation equipped with the new logarithmic contraction.

## 6. Conclusions

The application of the contraction with the logarithmic profile improves the flow at the inlet of the conduit, ensuring a pressure loss of almost zero and an approximately uniform velocity profile, which allows the measurement of the inlet velocity of the flow at best conditions, both through static and total pressure differences, and through hot element anemometers. The uniformity of the inlet profile is also beneficial in making comparable assays of different intercoolers. The permanent use of contraction will add great value to future trials, no matter the *intercooler* configuration.

The use of hot element anemometers has been shown to improve the sensitivity of the measurement especially at low speeds. The efficacy of this method can be greatly enhanced by a properly performed calibration in an appropriate installation and the use of an acquisition system with a resolution of 12 bits or greater.

In the "cold" tests the speed measured by the different methods have a difference between them of less than 0.4 m/s for speeds of less than 5m/s and maximum of 0,6 m/s for the remaining speeds. It is not possible to conclude anything regarding the accuracy of measurements. This would require a more rigorous calibration of hot the element anemometer.

The best solution for accurately measuring flow velocity will be a combination of the two methods, where the hot element anemometer is used to measure low speeds (less than 6m/s) and the calculation of speed through dynamic pressure at the other speeds (above 6m/s). Failure to use the anemometer at high speeds is due to the fact that turbulence levels are higher which translates into greater oscillation in measurements over the acquisition time. This may be because the stabilization chamber is not well designed, to the point where the turbulence dissipates to the inlet of the conduit, or the acquisition time of the hot element anemometer is not large enough.

Due to the difficulty of the immediate integration of the hot element sensors in the plant and connect them to the acquisition and control system used by JDEUS, it was not possible to test them for the presence of internal air flow. Thus, the best solution, for now, will be to replace the current measurement method by the calculation method through the dynamic pressure with the contraction mounted in the installation. This method, despite having an uncertainty of around 20% for low speeds, is more correct than the present one,

considering the heat exchanges between the internal air and the external air that have influence in the external pressure loss of an intercooler. In addition to improving the speed measurement, this measurement method is also easier, and pre-test measurements with the fan anemometer are not necessary.

## 7. Future Work

Construction of the calibration tunnel, which will allow a very good measurement of the hot element anemometers and use of a 12 bits or higher hot element anemometer signal acquisition system.

New tests, with both methods of velocity measurement addressed, for other configurations of intercoolers.

## References

- [1] Idel'chik, I. E. (1966). *Handbook of Hydraulic Resistance* (1st ed.).
- [2] Comte Bellot, G. (1976). Hot-Wire Anemometry Anemometry. *Annual Review of Fluid Mechanics*, 209–231.
- [3] King, L. V. (1914). On the Convection of Heat from Small Cylinders in a Stream of Fluid: Determination of the Convection Constants of Small Platinum Wires with Applications to Hot-Wire Anemometry. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 214(509–522), 373–432. <https://doi.org/10.1098/rsta.1914.0023>
- [4] Morel, T. (1977). Design of Two-Dimensional Wind Tunnel Contractions. *Journal of Fluids Engineering*, 99(2), 371. <https://doi.org/10.1115/1.3448764>
- [5] Bell, J. H., & Mehta, R. D. (1988). Contracton Design for Small Low Speed Wind Tunnels. *Joint Institute for Aeronautics and Acoustics - Stanford University*, (April).
- [6] Fang, F. M. (1997). A design method for contractions with square end sections. *Journal of Fluids Engineering*, 119(June 1997), 454–458. <https://doi.org/10.1115/1.2819156>
- [7] Fang, F.-M., Chen, J. C., & Hong, Y. T. (2001). Experimental and analytical evaluation of flow in a square-to-square wind tunnel contraction. *Journal of Wind Engineering and Industrial Aerodynamics*, 89(3–4), 247–262. [https://doi.org/10.1016/S0167-6105\(00\)00080-5](https://doi.org/10.1016/S0167-6105(00)00080-5)
- [8] Ramaseshan, S., & Ramaswamy, M. A. (2002). A Rational Method to Choose Optimum Design for Two-Dimensional Contractions. *Journal of Fluids Engineering*, 124(2), 544. <https://doi.org/10.1115/1.1456463>
- [9] Rodríguez Lastra, M., Fernández Oro, J. M., Galdo Vega, M., Blanco Marigorta, E., & Santolaria Morros, C. (2013). Novel design and experimental validation of a contraction nozzle for aerodynamic measurements in a subsonic wind tunnel. *Journal of Wind Engineering and Industrial Aerodynamics*, 118, 35–43. <https://doi.org/10.1016/j.jweia.2013.04.008>